Detecting Oil And Gas Using Sound Waves

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February 6, 2022

MISG Graduate Modelling Camp

Outline

Introduction

The Problem

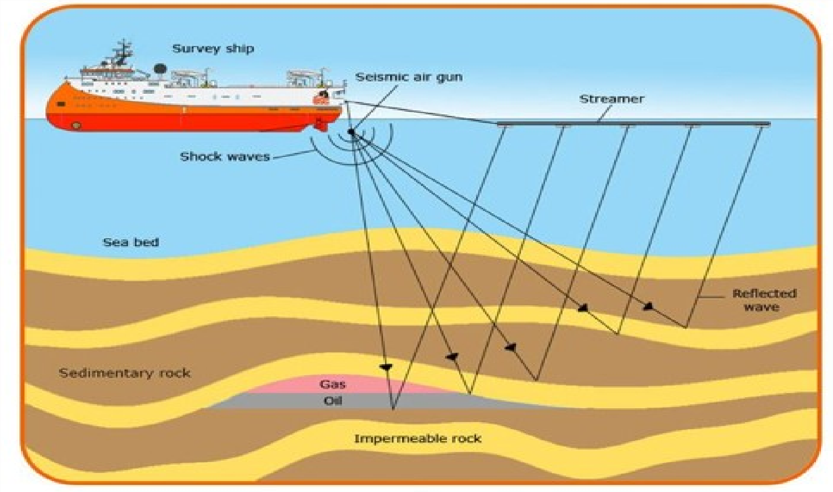


Figure 1: Detecting Gas and Oil using sound waves

Methods

Hooke's law

- The generalized Hooke's law is given by:

$$\tau_{ik} = C_{ikrs}E_{rs}. \tag{1}$$

  Where τ_{ik} is the stress tensor and C_{ikrs} is a constant.

- In the case of an elastically isotropic homogeneous material the generalized Hooke's law becomes:

$$\tau_{ik} = \lambda\theta\delta_{ik} + 2\mu E_{ik} \tag{2}$$

  where λ and μ can depend at most on temperature, and $\theta = E_{kk} = \nabla\cdot\underline{u}$

- The strain tensor is given by:

$$E_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right). \tag{3}$$

1D Wave Function

$$\frac{\partial^2 \phi}{\partial t^2} - V^2 \frac{\partial^2 \phi}{\partial x^2} = 0 \tag{4}$$

To resolve equation 4 for the position vector function $\phi(x, t)$, new variables $\xi = x - Vt$ and $\eta = x + Vt$ are introduced. Leading to ϕ being a direct function of ξ and η. And are only indirectly related to x and t. See equation (5)

$$\phi = \phi(\xi(x, t),\ \eta(x, t)) \tag{5}$$

WAVE FUNCTION CONTINUE

$$\frac{\partial \xi}{\partial x} = 1,\ \frac{\partial \xi}{\partial t} = -V,\ \frac{\partial \eta}{\partial x} = 1,\ \frac{\partial \eta}{\partial t} = V \tag{6}$$

Where f and g are arbitrary functions determined by the initial, the boundary conditions and expressed in terms of x and t. See equation (7)

$$\phi(x,t) = f(x - Vt) + g(x + Vt) \tag{7}$$

Theoretical Example

For simplicity we first consider an ocean with an indefinite depth, see figure 2

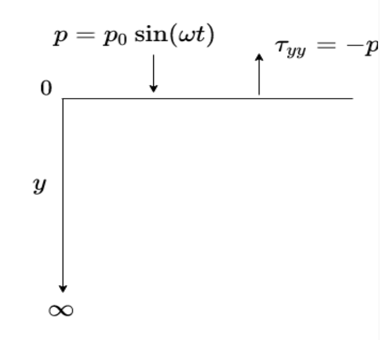


Figure 2: Simplified case

Example Problem Boundary Conditions

In this case,

$$u_y(y,t) = f(y - v_p t) \tag{8}$$

With boundary conditions:

- Initially the material is stationary:

$$\frac{\partial u_y}{\partial t}(y,0) = 0 \tag{9}$$

- Pressure at $y = 0$:

$$\tau_{yy}(0,t) = -p_0 sin(\omega t) \tag{10}$$

- At $t = 0$ and $y = 0$ there is no displacement:

$$u_y(0,0) = 0 \tag{11}$$

Wave Propagating Function

Solving equation (4) with $u_y(y,t) = f(y - v_p t)$ and boundary conditions (9),(10),(11) we get;

$$U_y(y,t) = f(y - V_p t) = \begin{cases} \frac{\frac{P_0 \omega}{V_p}}{\lambda + \mu} \left(cos(\frac{\omega}{V_p}(y - V_p t) \right), & \text{if} \quad y \leq V_p t \\ 0, & \text{otherwise} \quad y \geq V_p t \end{cases} \tag{12}$$

Simulation of Example Results

Propagation Direction

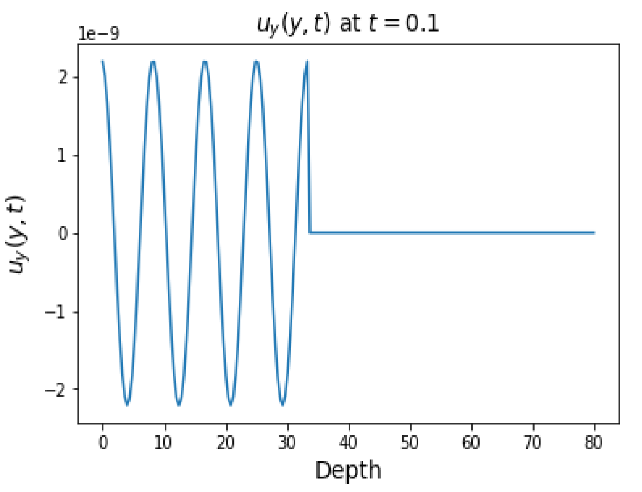


Figure 3: Depth at $t = 0.1$

Propagation Direction

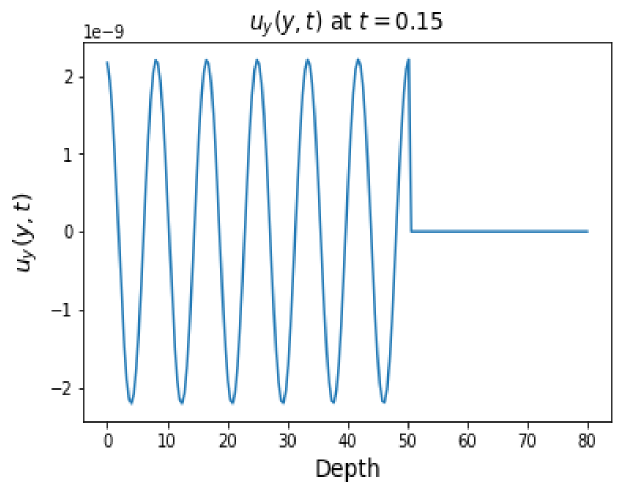


Figure 4: Depth at $t = 0.15$

Propagation Direction

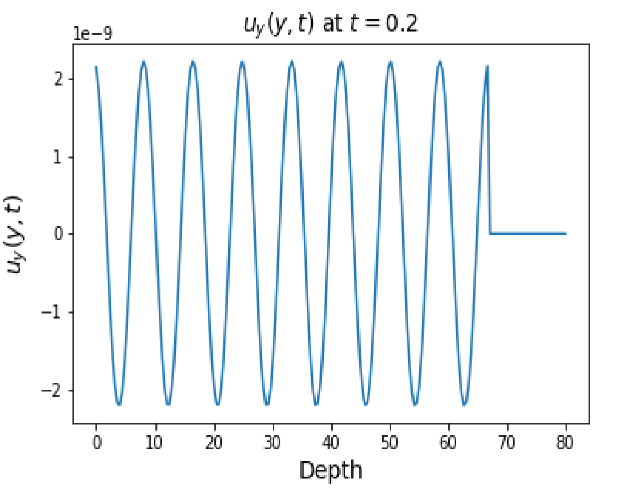
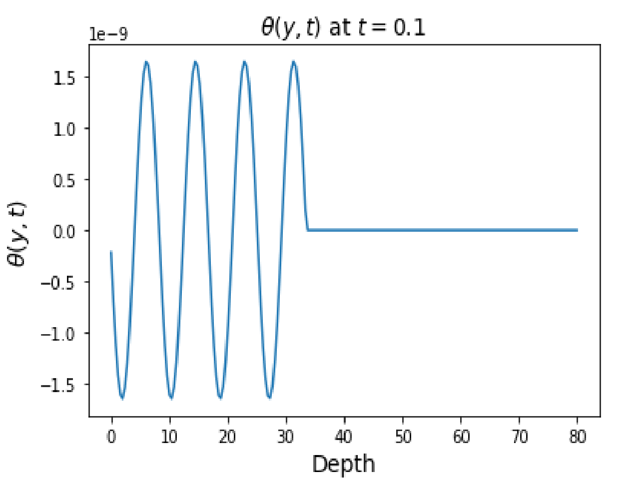
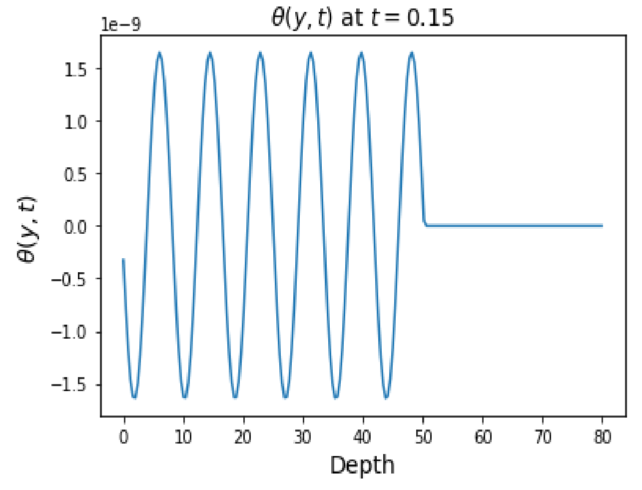


Figure 5: Depth at $t = 0.2$

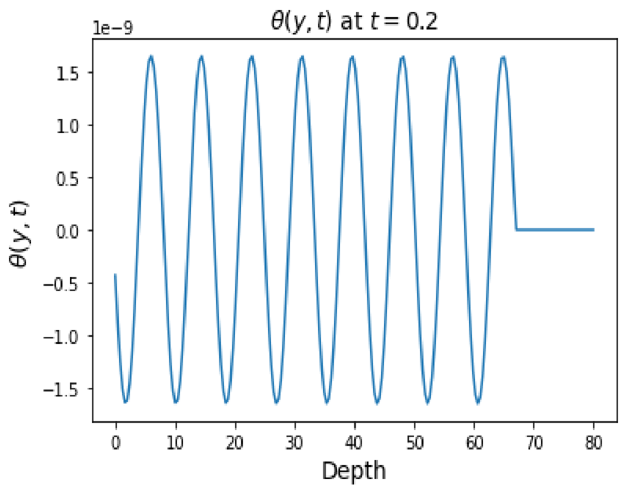
Dilation simulation



Dilation simulation



Dilation simulation



Application to the Problem

1D Consideration

We now consider a more realistic scenario, see figure 2

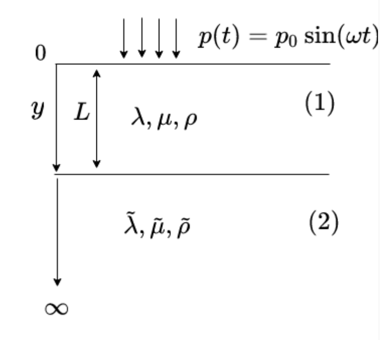


Figure 6: Application to the oil and gas detection problem

Boundary Conditions

For the case $0 \leq t \leq t^*$,

$$u_y(y,t) = f(y - v_p t) \tag{13}$$

With boundary conditions:

- Initially the material is stationary:

$$\frac{\partial u_y}{\partial t}(y,0) = 0 \tag{14}$$

- Pressure at $y = 0$:

$$\tau_{yy}(0,t) = -p_0 sin(\omega t) \tag{15}$$

- At $t = 0$ and $y = 0$ there is no displacement:

$$u_y(0,0) = 0 \tag{16}$$

Wave Propagating Function

Solving equation (4) with $u_y(y,t) = f(y - v_p t)$ and boundary conditions (14),(15),(16) we get;

$$U_y(y,t) = f(y - V_p t) = \begin{cases} \frac{P_0 V_p}{\omega(\lambda+2\mu)} \left(1 - \cos(\frac{\omega}{V_p})\right), & \text{if} \quad V_p(t - t^*) \leq y \leq V_p t \\ 0, & \text{otherwise} \quad V_p t \leq y \leq L \end{cases} \tag{17}$$

Reflected Wave

When $t \geq t^*$
There will be reflection from the boundary at $y = L$

$$u_y(y,t) = f(y - v_p t) + g(y + v_p t)$$

Now we have to consider the boundary at $y = L$ After the wave reaches the boundary:

$$\tilde{u}_y = \tilde{f}(y - \tilde{v}_p t)$$

Reflected wave

Continuity of stress

Stress in the upper layer = stress in the lower layer

$$\tau_{yy}(L,t) = \tilde{\tau_{yy}}(L,t)$$

$\therefore$

$$(\lambda + 2\mu)\frac{\partial u_y}{\partial y} = (\tilde{\lambda} + 2\tilde{\mu})\frac{\partial \tilde{u_y}}{\partial y}$$

$\therefore$

$$(\lambda + 2\mu)\left[f'(L - v_p t) + g'(L - v_p t)\right] = (\tilde{\lambda} + 2\tilde{\mu})\tilde{f}'(L - \tilde{v_p} t) \qquad (18)$$

Integrating the above equation, we have:

$$(\lambda + 2\mu)\left[-1/v_p f(L - v_p t) + 1/v_p g(L + v_p t)\right] = -\frac{(\tilde{\lambda} + 2\tilde{\mu})}{v_p}\tilde{f}(L - \tilde{v_p} t) \qquad (19)$$

Reflected wave

Add the boundary condition:

Continuity of displacement at $y = L$ (NO gaps)

$$u(L,t) = \tilde{u}(L,t)$$

$\implies$

$$f(L - v_p t) + g(L + v_p t) = \tilde{f}(L - \tilde{v_p} t) \tag{20}$$

Our goal is to find the reflected wave.

$$g(L - v_p t)$$

Reflected wave

Solving equation 19 and equation 20 simultaneously, we get that the reflected wave is given by:

$$g(y + v_p t) = -\frac{\left[1 - \frac{\tilde{v_p}(\lambda+2\mu)}{v_p(\tilde{\lambda}+2\tilde{\mu})}\right]}{\left[1 + \frac{\tilde{v_p}(\lambda+2\mu)}{v_p(\tilde{\lambda}+2\tilde{\mu})}\right]} \frac{P_0 v_p}{\lambda + 2\mu} \left[1 - \cos(\omega/v_p(2L - (y + v_p t))\right]$$

What we are interested in is the information that we get from the reflected wave

Discussion

Discussion

- Cauchy's first law of motion
- generalised Hooke's law for elastic materials
- Wave equation in 1-D
- Interested in longitudinal wave
- Solutions of the longitudinal wave
- Reflected wave that gives the information about the underlying rocks

Conclusion

Conclusion

The model works perfectly well, one will need to know the parameters that give the amplitude and the phase of the reflected wave.
The model can be improved by working on a 2-D, where we consider both the reflection and the refracted part of the wave

Thank you!